Math 32BH
Midterm 2 Solutions
Feb 28, 2005

1. (25 points) Compute the improper integral

$$\int_{\mathbb{R}} x^2 e^{-x^2} dx$$

(Show detailed work to get full credit.)

Use integration by parts. Let $u = x$, $du = dx$, $dv = xe^{-x^2}dx$, $v = -\frac{1}{2}e^{-x^2}$. Then $\int_{\mathbb{R}} x^2 e^{-x^2} dx = \frac{1}{2}\int_{\mathbb{R}} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ by the trick shown in class.

2. (25 points) Compute the line integral $\int_\gamma \overrightarrow{F} \cdot d\overrightarrow{x}$ where

$$\overrightarrow{F} = (-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2})$$

and γ is the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ oriented counterclockwise. (Show detailed work to get full credit.)

$\int_\gamma \overrightarrow{F} \cdot d\overrightarrow{x} = 2\pi$. See example 2.7 in the text for the details.

3. (25 points) Compute $div(\overrightarrow{F})$ where $\overrightarrow{F} = \frac{\overrightarrow{x}}{|\overrightarrow{x}|^3}$ is a vector function from $\mathbb{R}^3$ to $\mathbb{R}^3$.

By calculation $div(\overrightarrow{F}) = 0$.

4. (1). (15 points) Let $\overrightarrow{F} = (6xy - y^3, 4y + 3x^2 - 3xy^2)$. Determine whether $\overrightarrow{F}$ is conservative on $\mathbb{R}^3$. If it is, please find a potential function.

If $\frac{\partial p}{\partial x} = 6xy - y^3$, then $p(x,y) = 3x^2y - xy^3 + f(y)$, where f is some function of y alone. So if $\frac{\partial p}{\partial y} = 3x^2 - 3xy^2 + \frac{df}{dy} = 4y + 3x^2 - 3xy^2$, then $f(y) = 2y^2 + C$, where C is a constant. It follows $p(x,y) = 3x^2y - xy^3 + 2y^2 + C$.

(2). (10 points). Compute the line integral of the above vector field along the unit circle from $(1,0)$ to $(0,1)$.

The integral evaluates to $p(0,1) - p(1,0) = 2$.