Math 32BH
Homework 7 Solutions

I graded 4 of the problems:
Page 456: 2, 8, 12, 18.

The following are solutions to the homework problems and additional comments for the problems I graded. Note that solutions are often brief; if you need more detail please ask in section or office hours. I may well have made errors in my solutions so please let me know if I did. For grading information see my class webpage.

General Comments
The maximum number of points was 12. The high score was 12, the median was 11, and the mean was 10.4.

Page 456

2. Either $(x, y, \sqrt{x^2+y^2})$ with $x^2+y^2 \le 4$; or $(r\cos\theta, r\sin\theta, r)$, with $r \in [0,2]$ and $\theta \in [0, 2\pi]$ is fine.

5. One solution is in the back of the book.

8. Either (x, y, x^2+y^2) with $1 \le x^2+y^2 \le 4$; or $(r\cos\theta, r\sin\theta, r^2)$, with $r \in [1,2]$ and $\theta \in [0, 2\pi]$ is fine.

12. $\int_0^{2\pi}\int_0^2 \sqrt{1-4r^2}\, r\, dr\, d\theta = \frac{\pi}{6}\left(17\sqrt{17}-1\right)$.

18. Let D be the unit circle; then by symmetry the area is $4\iint_D \frac{1}{\sqrt{1-x^2}}\, dx\, dy = 8\int_0^1\int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2}}\, dy\, dx = 8\int_0^1 dx = 16$. Note that in section and office hours I missed part of the surface and was off by a factor of two. Thanks to Jared Warner for pointing this out, and sending a useful link to a picture of the surface:

    `http://astronomy.swin.edu.au/~pbourke/polyhedra/cylinders/plus2.jpg`

    I was looking directly at say the blue surface and missed the red surface. Jared also pointed out another way to solve the problem is to use cylindrical coordinates and parematerize half of one blue section as $(\cos u, \sin u, v)$ where $(u, v) \in [0, \pi] \times [0, \sin u]$; the surface area is easily evaluated to be 2, and multiplying by 8 you get the solution.

24. Assume $a, b, c > 0$. Then the area is

$$2\int_{-b}^{b}\int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} \sqrt{1 + \frac{c^2(b^4x^2 + a^4y^2)}{a^4b^4\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)}}\, dx\, dy.$$

    You can also use spherical coordinates.