Math 32BH
Homework 5 Solutions

I graded 4 of the problems:
Page 411: 8, 14;
Page 424: 4, 25.

The following are solutions to the homework problems and additional comments for the problems I graded. Note that solutions are often brief; if you need more detail please ask in section or office hours. I may well have made errors in my solutions so please let me know if I did. For grading information see my class webpage.

General Comments
The maximum number of points was 12. The high score was 12, the median was 10, and the mean was 9.9.

Page 411

2. $\int_0^1 \left(t + (1-t^2) - 3t\right)\, dt = -\frac{1}{6}$
7. Solution is in the back of the book.
8. -2π
11. Solution is in the back of the book.
14. Parameterize the curve (a circle) by $x = t$, $y = 2 - t$, and $z = \pm\sqrt{4t - 2t^2}$, where t goes from 0 to 2 in the upper half circle and from 2 to 0 in the lower half circle. Combine the two integrals, do a trig substitution and you should get $-2\sqrt{2}\pi$.
17. Solution is in the back of the book.
25. Solution is in the back of the book.

Page 424

2. $\frac{3\pi}{2}$
4. $\int_{-1}^1 \int_{-1}^1 (e^y + e^x)\, dx\, dy = 4e - \frac{4}{e}$
10. $\int_0^1 y \int_1^2 \, dx\, dy = \frac{1}{2}$
12. -2π
15. Solution is in the back of the book.
25. If 0 is not enclosed by γ then P and Q satisfy the conditions of Theorem 2.3 and the integral is 0. Otherwise let γ' be a circle of radius $\epsilon > 0$ entirely contained within γ, oriented clockwise, and follow the argument of Example 2.7 to calculate the integral to be 2π. Note that I expected you to show all the work to get full credit.
26. It is an easy calculation that $|\mathbf{N}| = 1$. Note that we assume that $\mathbf{x}'(t) \neq 0$ for all $t \in [a, b]$. Also $\mathbf{x}'(t) \cdot \mathbf{N}(t) = \frac{1}{|\mathbf{x}'(t)|}\left(\frac{dx}{dt}\frac{dy}{dt} - \frac{dy}{dt}\frac{dx}{dt}\right) = 0$.
28. We have $\iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA = \int_\gamma -Q\, dx + P\, dy = \int_\gamma \mathbf{F} \cdot \mathbf{N}\, ds$.