# Math 32BH
## Homework 4 Solutions

I graded 4 of the problems:
Page 306: 16;
Page 313: 24;
Page 323: 24, 27.

The following are solutions to the homework problems and additional comments for the problems I graded. Note that solutions are often brief; if you need more detail please ask in section or office hours. I may well have made errors in my solutions so please let me know if I did. For grading information see my class webpage.

**General Comments**
The maximum number of points was 12. The high score was 12, the median was 11, and the mean was 9.

### Page 306

2. $M = \int_0^\pi \int_0^a d\,r\,dr\,d\theta = \frac{da^2\pi}{2}$. $\bar{x} = 0$ by symmetry. $\bar{y} = \frac{d}{M}\int_0^a r^2 \int_0^\pi \sin\theta\,d\theta\,dr = \frac{4a}{3\pi}$.

12. By Pappus's Theorem we have $V = 2\pi a \frac{\pi a^2}{2} = \pi^2 a^3$.

16. $M = k\int_0^3 \rho^3 \int_0^{\frac{\pi}{2}} \sin\phi \int_0^{2\pi} d\theta\,d\phi\,d\rho = \frac{81k\pi}{2}$. $\bar{x} = \bar{y} = 0$ by symmetry.
$\bar{z} = \frac{k}{M}\int_0^3 \rho^4 \int_0^{\frac{\pi}{2}} \sin\phi\cos\phi \int_0^{2\pi} d\theta\,d\phi\,d\rho = \frac{243k\pi}{5} \cdot \frac{2}{81k\pi} = \frac{6}{5}$.

### Page 313

5. Both equal $(1+y)^5 - y^5$.

10. $F'(x) = \frac{\cos\pi x - \cos\frac{\pi x}{2}}{x}$.

21. Although one can use integration by parts to evaluate this integral, to use Leibniz's Rule evaluate both sides of $\frac{d}{da}\int_\epsilon^1 x^a\,dx = \int_\epsilon^1 \frac{\partial}{\partial a}x^a\,dx$. Note that $y$ should be $a$ in the problem.

24. Taking the derivative by $a$ of both sides of $\int_0^a \frac{dx}{(x^2+a^2)^2} = \frac{1}{4a^3} + \frac{\pi}{8a^3}$, we get
$\int_0^a \frac{-4a\,dx}{(x^2+a^2)^3} + \frac{1}{4a^4} = \frac{-3}{4a^4} + \frac{-3\pi}{8a^4}$, and so $\int_0^a \frac{dx}{(x^2+a^2)^3} = \frac{1}{4a^5} + \frac{3\pi}{32a^5}$.

### Page 323

2. Converges to $\frac{\pi}{2}$.

8. Converges to $\pi$.

13. Converges to $2\pi$.

18. Diverges to $+\infty$.

24. First note that $n(x) \geq 0$ for all $x$. Now
$\left(\int_0^\infty n(x)\,dx\right)^2 = \frac{1}{2\pi}\int_0^\infty\int_0^\infty e^{\frac{-(x^2+y^2)}{2}}\,dx\,dy = \frac{1}{2\pi}\int_0^{\pi/2}\int_0^\infty re^{\frac{-r^2}{2}}\,dr\,d\theta = \frac{1}{4}$, and so $\int_0^\infty n(x)\,dx = \frac{1}{2}$. As you can see there is a typo in the book and the original interval of integration should have been $(-\infty,\infty)$. In this case we get
$\left(\int_{-\infty}^\infty n(x)\,dx\right)^2 = \frac{1}{2\pi}\int_{-\infty}^\infty\int_{-\infty}^\infty e^{\frac{-(x^2+y^2)}{2}}\,dx\,dy = \frac{1}{2\pi}\int_0^{2\pi}\int_0^\infty re^{\frac{-r^2}{2}}\,dr\,d\theta = 1$, and so $\int_{-\infty}^\infty n(x)\,dx = 1$.
I gave full credit for either solution as long as you were clear and consistent.

27. (a) Note that we must have $a > 0$. The base case for $n = 0$ is easy. Assume true for $n-1$ and prove for $n$. We have $\int_0^\infty x^n e^{-ax}\,dx = \left(\frac{-1}{a}x^n e^{-ax}\right)\Big|_0^\infty + \frac{n}{a}\int_0^\infty x^{n-1}e^{-ax}\,dx = \frac{n}{a}\frac{(n-1)!}{a^n} = \frac{n!}{a^{n+1}}$.

    (b) $\frac{c^3}{32\pi}\int_0^{2\pi}\int_0^\pi \sin\phi\int_0^\infty \rho^2(1-c\rho)^2 e^{-c\rho}\,d\rho\,d\phi\,d\theta = \frac{c^3}{8}\int_0^\infty \rho^2(1-c\rho)^2 e^{-c\rho}\,d\rho = \frac{7}{4}$. Note that there is a typo in the problem (what else is new?). Jared Warner looked up the correct formula and found the $(1-c\rho)$ should actually be $(2-c\rho)$. In this case you should get 1.