Math 32BH
Homework 1 Solutions

I graded 4 of the problems:
Page 238: 6, 15;
Page 245: nothing!;
Page 254: 16, 28.

The following are solutions to the homework problems and additional comments for the problems I graded. Note that solutions are often brief; if you need more detail please ask in section or office hours. I may well have made errors in my solutions so please let me know if I did. For grading information see my class webpage.

General Comments
The maximum number of points was 12. The high score was 12, the median was 9.7, and the mean was 10.

Page 238

2. $\iint_R e^{x+y}\,dA = \left(\int_0^1 e^x\,dx\right)\left(\int_0^2 e^y\,dy\right) = (e-1)(e^2-1)$.

6. $\iint_R (\sec^2 x \sin y - \sin y)\,dA = \left(\int_0^{\pi/4} (\sec^2 x - 1)\,dx\right)\left(\int_{\pi/6}^{\pi/3} \sin y\,dy\right) =$
$\left((\tan x - x)|_0^{\pi/4}\right)\left(-\cos y|_{\pi/6}^{\pi/3}\right) = (1-\frac{\pi}{4})\frac{\sqrt{3}-1}{2}$.

12. $\iint_R y \ln xy\,dA = \left(\int_1^2 \ln x\,dx\right)\left(\int_2^3 y\,dy\right) + \left(\int_1^2 dx\right)\left(\int_2^3 y \ln y\,dy\right) =$
$(2\ln 2 - 1)(5/2) + \frac{9}{2}\ln 3 - \frac{9}{4} - 2\ln 2 + 1$.

13. If $f(x,y) \geq 0$ for all $(x,y) \in R$, then $R(f,G,z) \geq 0$ for all G and z, and therefore $\lim_{|G|\to 0} R(f,G,z) \geq 0$ as well, which gives the result.

14. Apply exercise 13 to the function $(g-f)(x,y) = g(x,y) - f(x,y)$ and use linearity.

15. Let $g(x,y) = M$ for all x, y; then exercise 14 gives $\iint_R f(x,y)\,dA \leq \iint_R M\,dA = MA(R)$; the other inequality is similar.

18. Both the rational and irrational points are dense in $[0,1]$, which means that no matter how small an interval we pick we can find both a rational and an irrational number in it. It follows $\liminf_{|G|\to 0} R(f,G,z) \leq 1$ while $\limsup_{|G|\to 0} R(f,G,z) = 2$, and therefore the limit cannot exist.

22. Note that f must be assumed to be integrable (otherwise we could come up with an counterexample similar to exercise 18). If we use a uniform grid G that is symmetric about the y axis then we can always pick our points in each pair of symmetric subrectangles to be (x,y) and $(-x,y)$. Then the corresponding terms in the Riemann sum will cancel, and so the integral is 0.

Page 245

2. $\int_1^4 \int_{-1}^2 (x^2 - 3x + 2y)\,dx\,dy = \int_1^4 (-\frac{3}{2} + 6y)\,dy = \frac{81}{2}$.

8. $\int_0^{\pi/2} \int_0^{\pi/2} \cos(x+y)\,dx\,dy = \int_0^{\pi/2} (\sin(y+\pi/2) - \sin(y)) = 2$.

12. $\int_1^2 \int_0^2 \frac{1}{x+y}\,dy\,dx = \int_1^2 (\ln(x+2) - \ln(x))\,dx = 4\ln 4 - 3\ln 3 - 2\ln 2$.

14. $\int_0^1 \int_0^2 16 - x^2 - y^2\,dx\,dy = \int_0^1 32 - \frac{8}{3} - 2y^2\,dy = 32 - \frac{10}{3}$.

18. $\int_1^2 \int_0^1 ye^{xy}\,dx\,dy = \int_1^2 (e^y - 1)\,dy = e^2 - e - 1$.

19. Part (a) follows directly from Fubini's Theorem. In (b), the approximation for I follows by plugging the approximation for $g(y)$ into the definition of I and applying the trapezoidal rule.

20. The exact value is $\frac{7}{6} \approx 1.17$. Working out the approximate value is tedious and I haven't done it yet. There are 16 squares to sum over.

Page 254

2. $\int_0^1 \int_0^{3x} (2 - 3x + xy)\, dy\, dx = \int_0^1 (6x - 9x^2 + \frac{9}{2}x^3)\, dx = \frac{9}{8}$.

4. $\int_{-1}^1 \int_{y^2}^1 (x - y^2)\, dx\, dy = \int_{-1}^1 (\frac{1}{2} - y^2 + \frac{y^4}{2})\, dy = \frac{28}{15}$.

10. The function is symmetric in all four quadrants of the xy-plane, and so it suffices to evaluate $4\int_0^1 \int_0^{1-x} (1 + x^2 + y^2)\, dy\, dx = 4\int_0^1 (1 - 2x - 2x^2 - \frac{4}{3}x^3)\, dx = \frac{4}{3}$.

14. $\int_0^1 \int_{y^4}^{y^2} dx\, dy = \int_0^1 (y^2 - y^4)\, dy = \frac{2}{15}$.

16. By symmetry it suffices to evaluate $2\int_{1/2}^1 \int_0^{\sqrt{1-y^2}} dx\, dy = 2\int_{1/2}^1 \sqrt{1-y^2}\, dy = (\theta + \frac{1}{2}\sin 2\theta)\Big|_{\pi/6}^{\pi/2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$.

20. $\int_0^1 \int_0^2 (2 - \frac{x}{2} - \frac{y}{2})\, dy\, dx = \int_0^1 (3 - x)\, dx = \frac{5}{2}$.

24. $\int_0^1 \int_{x^3}^{x} xy\, dy\, dx = \int_0^1 \frac{1}{2}(x^3 - x^7)\, dx = \frac{1}{16}$.

28. $\int_0^2 \int_0^{y/2} \sqrt{4 - y^2}\, dx\, dy = \int_0^2 \frac{y}{2}\sqrt{4 - y^2}\, dy = \frac{4}{3}$.

30. The integral equals $\int_0^{\sqrt{2}} \int_{\sqrt{y}}^2 f(x, y)\, dx\, dy$.

34. The integral equals $\int_1^4 \int_{x-2}^{\sqrt{x}} f(x, y)\, dy\, dx + \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} f(x, y)\, dy\, dx$.