Math 115A
Homework 8 Comments

I graded 5 of the problems:
Section 6.1: 16b, 19a
Section 6.2: 2a, 2c, 4

Each problem is worth 2 points. A grade of 0 indicates no solution or a substantially wrong solution. A grade of 2 indicates a correct or nearly correct solution. Otherwise the grade given is 1.

If you believe a problem was misgraded, or I made some addition or other error, please write a short note explaining the situation, attach it to your homework, and return it to me (either in person, in my mailbox, or under my office door). I'll take a look and afterwards leave your homework in a box outside my office.

The following are comments and occasionally solutions for the graded problems.

General Comments
The maximum number of points was 10. The high score was 9. The mean was 5.9. A couple people would have gotten 10 if it weren't for my strict grading of problem 19a.

6.1

16b. I debated whether to grade this or 15(a), since I covered both of them in section, and decided on this one mainly because I didn't want to wade through all the calculations in 15(a). I was a little disappointed to find that a lot of people didn't seem to understand my explanation of this problem very well, so let me try again.

First of all, the first three properties of an inner product all hold for this definition, as you easily check. For "conjugate commutativity," note that f and g are both real valued functions, and so the integral is also real.

It is the fourth property that causes a problem. It is possible to find a continuous function $f : [0,1] \to \mathbb{R}$ such that $f \neq 0$ (that is f is not zero for all values in $[0,1]$; this is the 0 element of V) but $\int_0^{1/2} f(t)^2 dt = 0$. There are many ways to do this, but note that f must be continuous, so you can't just use a step function (say 0 on $[0,1/2]$ and 1 on $(1/2,1]$). A simple function that would work is $f(t) = 0$ for $0 \leq t \leq 1/2$ and $f(t) = t - 1/2$ for $1/2 \leq t \leq 1$. Note that f is continuous at all points of $[0,1]$, in particular at $1/2$. And also although f is not identically 0 on $[0,1]$, it is identically 0 on $[0,1/2]$, so $\int_0^{1/2} f(t)^2 dt = 0$ and this cannot be an inner product.

19a. I covered all the section 6.1 problems on this homework in section except 19(a), so I decided to grade it even though it was easy. In fact the problem is essentially solved in the text in the proof of Theorem 6.2(d). However I wanted people to explain just where the $\Re\langle x,y\rangle$ comes from, and I took a point off if you didn't do this. Perhaps some people understood this part, but it seems most did not, and very few actually tried to explain it. The point is this: If you have any complex number z, write it as $a+bi$ where a and b are real. Then $z + \bar{z} = (a+bi) + (a-bi) = 2a = 2\Re(z)$. Applying this to $\langle x,y\rangle + \langle y,x\rangle = \langle x,y\rangle + \overline{\langle x,y\rangle}$ (remember that $\langle x,y\rangle$ is just a complex number), we get the result.

6.2

2a. This was the only Gram-Schmidt problem assigned that didn't have the answer in the back, and everyone must have guessed I'd grade it because almost everyone attempted it, even if they didn't try any of the other problems. I don't want to go through the calculations here, which are similar to those in Example 4, but the orthogonal basis was $\{v_1, v_2, v_3\}$ where $v_1 = (1,0,1)$, $v_2 = (-\frac{1}{2}, 1, \frac{1}{2})$ and $v_3 = (\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$. The orthonormal basis was then $\{u_1, u_2, u_3\}$ where $u_1 = \frac{1}{\sqrt{2}}(1,0,1)$, $u_2 = \frac{\sqrt{2}}{\sqrt{3}}(-\frac{1}{2}, 1, \frac{1}{2})$ and $u_3 = \sqrt{3}(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$.

The Fourier coefficients of x are then $\frac{3}{\sqrt{2}}$, $\frac{\sqrt{3}}{\sqrt{2}}$, and 0 respectively. Most people who got this far also verified Theorem 6.5 as per the directions, which was a good check to see if you made a mistake. Of course finding where the mistake is is a different matter....

2c. Again I won't go through the tedious calculations, which are similar to those in Example 5. The answer is in the back of the book.

4. A couple people started this problem by finding an orthogonal basis for the space spanned by S, but then they didn't know how to proceed from there and as this doesn't represent any real progress toward the solution I didn't give any credit for this. A couple other people took a sort of cross product of the two vectors of S to get an orthogonal vector, and although this happened to work it didn't show that this (and its span) was the only thing in $S^{\perp}$, and so I took off a point.

   The way to approach this problem is to understand the definition of $S^{\perp}$: the set of all vectors in $x \in V$ (here $V = \mathbb{C}^3$) which are perpendicular to every vector in S. So let $x = (a, b, c)$ be an arbitrary vector in $\mathbb{C}^3$. Now if $\langle x, (1, 0, i)\rangle = 0$, then $a + c\bar{i} = a - ci = 0$. Notice the $\bar{i}$ here! Many people forgot this and I took off a point in the hopes that you won't forget again. If also $\langle x, (1, 2, 1)\rangle = 0$, then we must also have $a + 2b + c = 0$.

   This gives us 2 equations in 3 unknowns, and it's simple algebra to solve them. The first equation gives us $a = ci$, so plugging into the second we have $a + 2b + c = ci + 2b + c = 0$ which implies $b = \frac{-c(1+i)}{2}$. So the vectors in $S^{\perp}$ all have the form $(ci, \frac{-c(1+i)}{2}, c)$ for $c \in \mathbb{C}$. Choosing $c = 1$ we get a basis for this space $(i, \frac{-(1+i)}{2}, 1)$, which is the answer in the back of the book.