

# Math 110BH

## Take-home Midterm Grading Comments

The following are comments about grading for the take-home midterm. I graded most problems carefully but there were some I looked at a little less carefully or not every part of. These exceptions are noted. No solutions will be posted, but of course feel free to ask me or Prof. Elman about problems about which you still feel unclear. Almost everyone put in a lot of effort into the midterm and the scores were very high. There was one perfect score and the median was 91.5 out of 100.

### 1 Part I

1. Everyone who got this right essentially followed the hints given in the textbook on page 238. However most people either made mistakes when doing induction or else were too vague. I took off a point or two depending on the severity of the error. Three people didn't follow the textbook method, and all three of those proofs were wrong! However I gave 4 points for effort in this case.
2. The hard direction of (i) was worth 6 points, the easy direction 2 points and (ii) was worth 2 points. Most people got full points or very close. Note that the textbook (and many other sources) says that a zero-divisor must be non-zero, whereas 2(ii) relies on zero being treated as a zero-divisor. I asked Prof. Elman about this, and he said zero should indeed be a zero-divisor (he defines it this way in problem 1), and that even books that say zero-divisors must be non-zero later contradict themselves (as does Dummit and Foote, as their exercise on page 238, used in problem 1 above, does include zero as a zero-divisor as it is stated).
3. Almost everyone used a standard proof for part (i) [8 points], whether it was referenced or not. Most people got the easy derivation of part (ii) [2 points] from part (i).
4. Five points per part. I checked that your norms were correct, that you made some effort to show they were norms, and that your calculations seemed reasonable. Most people did well on this problem.
5. Almost everyone followed Euclid's theorem. I wasn't too picky about details for this one. Seven points for (i) and three points for (ii).

### 2 Part II

1. I gave 5 points for (c)  $\Rightarrow$  (b), or equivalent (which I graded carefully), and 5 points for the rest (just checked if effort was made; 2 points were for ii). Note that if (c)  $\Rightarrow$  (b) was wrong then (a)  $\Rightarrow$  (b) was likely wrong as well as the two are similar. Most people followed one of two standard proofs and did fine.
2. I just glanced at (i) to see if you did it; (ii) was graded somewhat carefully (I mostly concentrated on linear independence) and (iii) and (iv) less so. Points: 1, 5, 2, 2. Most people put a lot of work into this problem and did very well.
3. I didn't grade whether or not you proved  $End_R(M)$  is a ring; perhaps this was done in class or in any case it is straightforward. For (i) I only checked that you show every nonzero element of  $End_R(M)$  has an inverse; this was worth 5 points. Part (ii) was also worth 5 points. Almost everyone followed the standard proofs here and I was not careful checking details.
4. I gave 1 point for claiming  $E_n(R) \subset SL_n(R)$  if you didn't do anything else, but otherwise the whole grade was for  $SL_n(R) \subset E_n(R)$ . I was very impressed that almost everyone got either 9 or 10 points for this problem, and although most people modified to the SNF algorithm to use only type I operations, several people did do something closer to my own solution, which uses only the division algorithm and a form of Gaussian elimination. Great job!
5. I checked that you covered all of the important points and did all of the necessary work, but I didn't check every detail. Seven points for (i) and three points for (ii).