Test 1 Topics

Calculus Theory

January 26, 2011

1 Two Hard Theorems

The following statements of the two major theorems we have covered suffice to memorize.

Theorem 1 (Intermediate Value Theorem)**.** *Let f be continuous on $[a, b]$ and let c satisfy either $f(a) < c < f(b)$ or $f(a) > c > f(b)$. Then there exists some $x \in [a, b]$ such that $f(x) = c$.*

Theorem 2 (Extreme Value Theorem)**.** *Let f be continuous on $[a, b]$. Then there exists some $y \in [a, b]$ such that $f(y) \leq f(x)$ for all $x \in [a, b]$ and also some $z \in [a, b]$ such that $f(z) \geq f(x)$ for all $x \in [a, b]$.*

2 Chapter 7

2.1 You Need to Know This

1. The precise statement of some form of the Intermediate Value Theorem and the Extreme Value Theorem.

2. Counterexamples to the two theorems when some of the hypotheses are removed.

3. Techniques for proving generalizations of the basic forms of the IVT and EVT, as used in the proofs of Theorems 4 to 7.

4. The proof of Theorem 8, that every positive number has a square root. Know how to generalize the theorem for nth roots, and the distinction between n being even and odd.

5. Applications of the IVT and EVT as in problems 1–3.

6. The solutions to the problem set problems 4, 10, 11. Pay special attention to the Brouwer Fixed Point Theorem and understand both its proof and meaning.

2.2 You Don't Need to Know This

1. Theorems 9, 10 and 11. You should know the statement of Theorem 9 however and how to use it (for example in problem 4).

3 Chapter 8

3.1 You Need to Know This

1. The definitions bounded above and below, uppper and lower bound, least upper bound and greatest lower bound. Know that sup is lub and inf is glb. Know a precise statement of the lub property (P13).

2. Know the statements and proofs of Theorems 2 and 3 in this chapter.

3. Be able to find lubs and glbs of sets, as in problem 1. Also be able to reason about them as in problem 12.

4. Know the meaning of density and how to prove the irrational and rational numbers are dense (problems 5 and 6 on the problem set).

3.2 You Don't Need to Know This

1. The proofs of the three hard theorems. Also the statement and proof of Theorem 1 of this chapter.